1 Cost function

Some explanations of the cost function used in the ActionModel class. The cost computation is divided in two : one relative to the state and one to the ground reaction forces : the command.

1.1 Cost on the state vector

w_s is a weight vector used to give more importance to certain parameters of the system's state. The residual cost is written :

$$w_s^T = \begin{bmatrix} \delta_1 & \dots & \delta_n \end{bmatrix}, \qquad R_x = w_s \begin{bmatrix} x_1 - x_{1,ref} & & \\ & \ddots & \\ & & x_n - x_{n,ref} \end{bmatrix}$$

$$cost(x)_t = \frac{\delta_1^2}{2}||x_1 - x_{1,ref}||^2 + \dots + \frac{\delta_n^2}{2}||x_n - x_{n,ref}||^2 = \frac{1}{2}R_x^T R_x \tag{1}$$

The partial derivative of the state cost can be written as :

$$L_x = R_x;$$

$$L_{xx} = \begin{bmatrix} \delta_1^2 & & \\ & \ddots & \\ & & \delta_n^2 \end{bmatrix} \tag{2}$$

1.2 Cost on the command vector

The cost function related to the command, is penalising when the friction cone is not respected. It is based on the cost function computed in the croccodyl.CostModelContactFrictionCone class. The command vector is $u^T = \begin{bmatrix} f_{x1} & f_{y1} & f_{z1} \dots & f_{xn} & f_{yn} & f_{zn} \end{bmatrix}$ where f_{x1} is the ground reaction force among the x-axis in the local frame of the first foot. The following constraints need to be respected :

$$|f_x| < \mu f_z \qquad |f_y| < \mu f_z \qquad f_z > 0 \tag{3}$$

The friction cone is a discrete approximation of the cone friction with 4 facets. For each foot the following residual vector is computed :

$$r = Au_i = \begin{bmatrix} 1 & 0 & -\mu \\ -1 & 0 & -\mu \\ 0 & 1 & -\mu \\ 0 & -1 & -\mu \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} f_{xi} \\ f_{yi} \\ f_{zi} \end{bmatrix}$$

Then, the activation function crocoddyl.ActivationModelQuadraticBarrier is used on the vector residual $r = Au_i$ to compute the cost relative to one friction cone $a_i(r)$, in odrer to penalises if the inequalities (3) are not respected.

$$a_i(r) = \frac{1}{2}||r_1^+||^2 + \dots + \frac{1}{2}||r_5^+||^2 = \frac{1}{2}||r^+||^2 \tag{4}$$

where $y^+ = y$ if y $>$ 0 and 0 if otherwise.

This function compute also the Jacobian A_r and the Hessian A_{rr} of the residual such as :

$$A_{r,i}^T = \begin{bmatrix} r_1^+ & \dots & r_5^+ \end{bmatrix}$$

$$A_{rr,i} = \begin{bmatrix} 1_I(r_1) & & \\ & \ddots & \\ & & 1_I(r_5) \end{bmatrix}$$

where $1_I(y) = 1$ if y > 0 and 0 otherwise.

The cost function relative to one friction cone is $a_i(r)$. The partial derivative of this cost can be written as :

$$L_{u,i} = A^T A_{ri} \qquad => \qquad L_u = \begin{bmatrix} L_{u1} & \dots & L_{u5} \end{bmatrix}$$

and the Hessian :

$$L_{uu,i} = A^T A_{rr,i} A \qquad => \qquad L_{uu} = \begin{bmatrix} L_{uu,1} & & \\ & \ddots & \\ & & L_{uu,5} \end{bmatrix}$$

To concluded, as this cost is computed for each foot, the final cost is and can be adjusted by the weight vector w_s and the weight on the command δ_u :

$$cost_t(x, u) = \frac{1}{2} R_x^T R_x + \delta_u \sum_{k=1}^{4} a_i$$

1.3 Graphical study

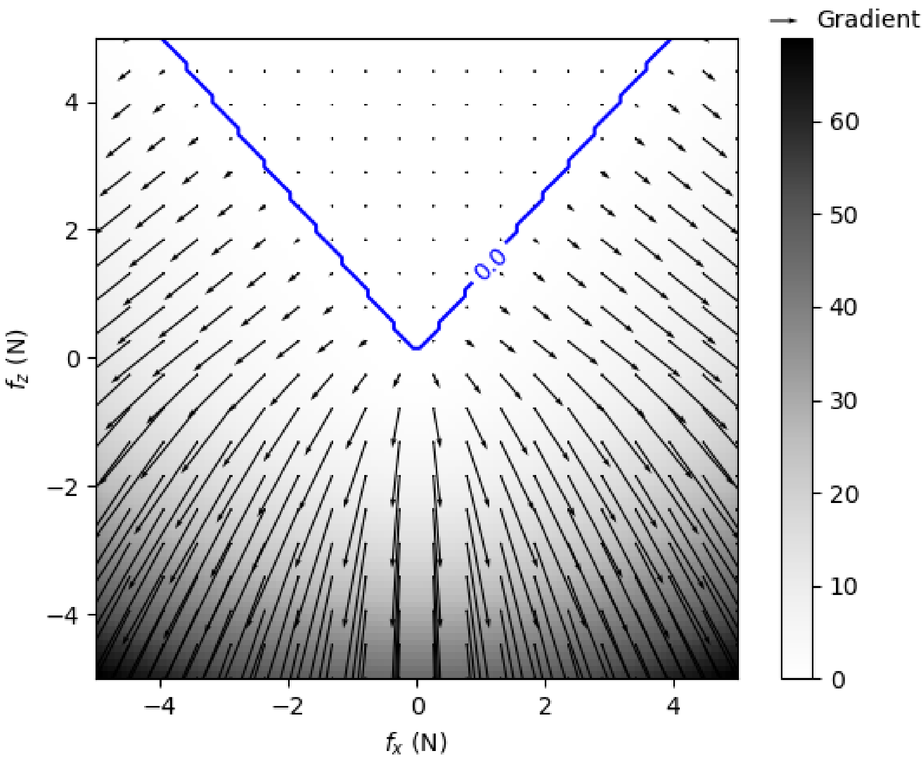

FIGURE 1 – Cost function wrt f_x and f_z , $f_y = 0$

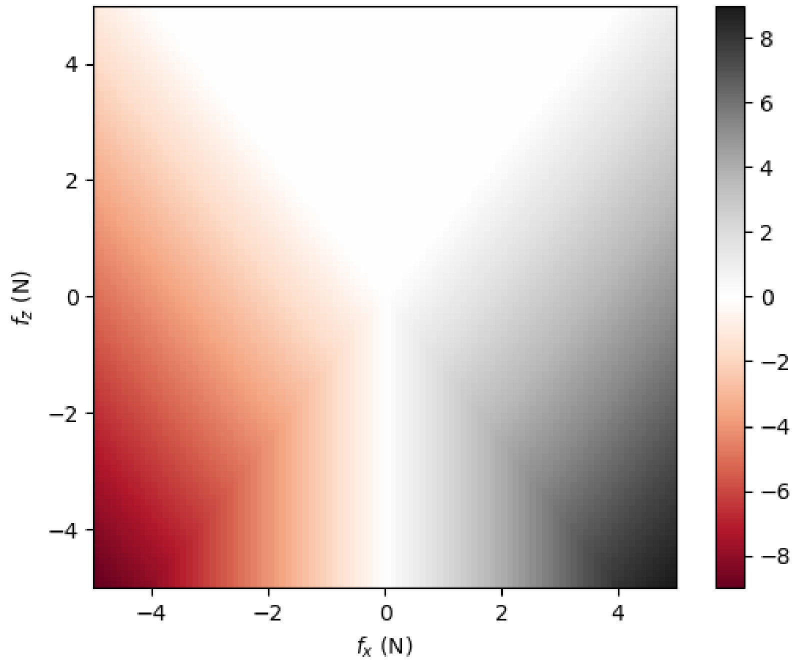

FIGURE 2 – $\frac{\partial Cost}{\partial f_x}(f_x, f_z), \quad f_y = 0$

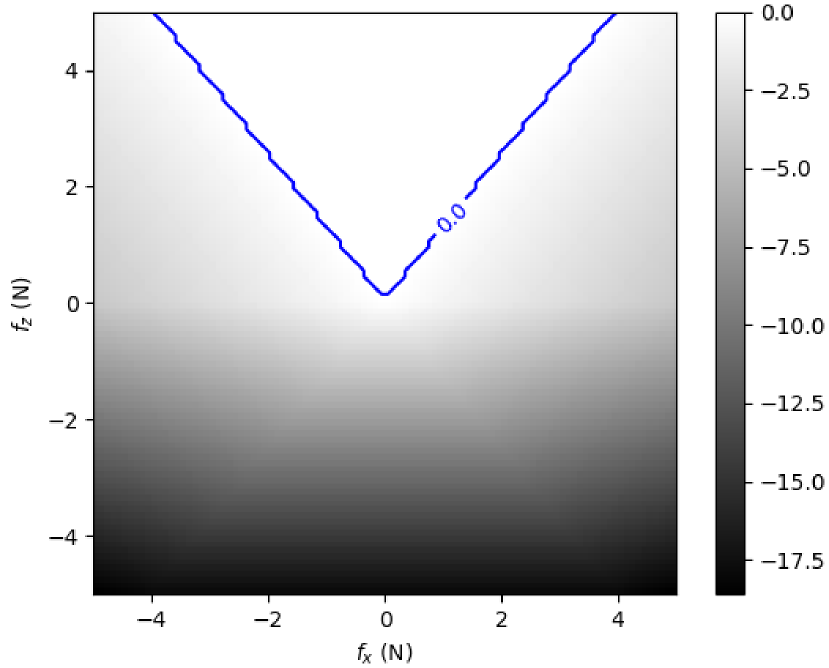

FIGURE 3 – $\frac{\partial Cost}{\partial f_z}(f_x, f_z), \quad f_y = 0$